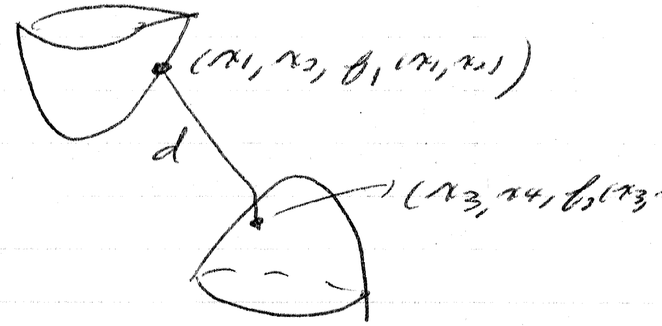
Q 6 [10 pts]



Sol.

Il faut minimiser la distance entre les 2 surfaces

$$\min d \Leftrightarrow \min d^2$$

$$d^2 = (x_1 - x_3)^2 + (x_2 - x_4)^2 + (f_1 - f_2)^2$$

où $f_1 = 1 + x_1^2 + x_2^2$

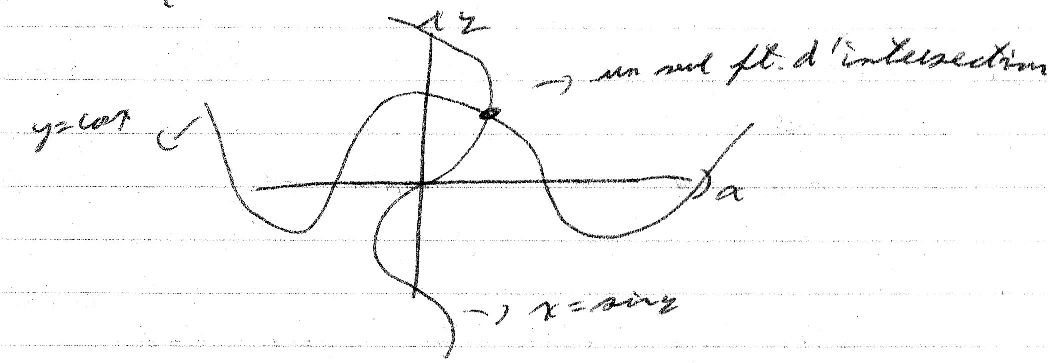
$f_2 = -(x_3 - 1)^2 - (x_4 - 1)^2$

$$F(x_1, x_2, x_3, x_4) = (x_1 - x_3)^2 + (x_2 - x_4)^2 + \left(1 + x_1^2 + x_2^2 + (x_3 - 1)^2 + (x_4 - 1)^2\right)^2$$

Q 7.

~~4~~. [35 pts]

a) [4 pts]



b) [6 pts]

$$J = \begin{array}{cc} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ \end{array}$$
$$J = \begin{pmatrix} 1 & -\cos y \\ \sin x & 1 \end{pmatrix}$$

2/

1e $\begin{pmatrix} 1 & -\cos y_k \\ \sin x_k & 1 \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \begin{pmatrix} \sin y_k - x_k \\ -y_k + \cos x_k \end{pmatrix}$ 2/ 1/ (5/)

2e) $x_{k+1} = x_k + \Delta x$ (1)
$y_{k+1} = y_k + \Delta y$

c) [4 pts]

Non 1/ car $\det J = 1 + \sin x \cos y = 0$ 1/
si $x = -\frac{\pi}{2}$, $y = 0$ 2/

d) [8 pts]

$$\begin{pmatrix} 1 & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 1 \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} - \frac{\pi}{4} \\ \frac{\sqrt{2}}{2} - \frac{\pi}{4} \end{pmatrix}$$

$$\Delta x = \frac{\left(\frac{\sqrt{2}}{2} - \frac{\pi}{4}\right) \begin{vmatrix} 1 & -\frac{\sqrt{2}}{2} \\ 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 1 \end{vmatrix}} = \left(\frac{\sqrt{2}}{2} - \frac{\pi}{4}\right) \frac{1 + \frac{\sqrt{2}}{2}}{3/2} \cong -.089$$

$$\Delta y = \frac{\left(\frac{\sqrt{2}}{2} - \frac{\pi}{4}\right) \begin{vmatrix} 1 & 1 \\ \frac{\sqrt{2}}{2} & 1 \end{vmatrix}}{|\quad|} = \left(\frac{\sqrt{2}}{2} - \frac{\pi}{4}\right) \frac{1 - \frac{\sqrt{2}}{2}}{3/2} \cong .015$$

$$x_1 = \frac{\pi}{4} + \Delta x \cong .696$$

$$y_1 = \frac{\pi}{4} + \Delta y \cong .770$$

$$\begin{pmatrix} & \end{pmatrix}^{-1} = \begin{pmatrix} 2/3 & \frac{\sqrt{2}}{3} \\ -\frac{\sqrt{2}}{3} & 2/3 \end{pmatrix}$$

e) [4 pts]

Si $\begin{cases} f(\bar{x}, \bar{y}) = 0 \\ g(\bar{x}, \bar{y}) = 0 \end{cases} \Rightarrow F(\bar{x}, \bar{y}) = 0$

Or $F(x, y) \geq 0 = F(\bar{x}, \bar{y}) \quad \forall (x, y) \in \mathbb{R}^2$

$\Rightarrow (\bar{x}, \bar{y})$ est un minimum de F

0) [9 pts]

$$F_x = (x - \sin y) + (y - \cos x)(\sin x)$$

$$F_y = (x - \sin y)(-\cos y) + (y - \cos x)$$

$$F_x\left(\frac{\pi}{4}, \frac{\pi}{4}\right) = \left(\frac{\pi}{4} - \frac{\sqrt{2}}{2}\right) + \left(\frac{\pi}{4} - \frac{\sqrt{2}}{2}\right)\frac{\sqrt{2}}{2}$$

$$= \left(\frac{\pi}{4} - \frac{\sqrt{2}}{2}\right)\left(1 + \frac{\sqrt{2}}{2}\right)$$

$$F_y\left(\frac{\pi}{4}, \pi/4\right) = \left(\frac{\pi}{4} - \frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(\frac{\pi}{4} - \frac{\sqrt{2}}{2}\right)$$

$$= \left(\frac{\pi}{4} - \frac{\sqrt{2}}{2}\right)\left(1 - \frac{\sqrt{2}}{2}\right)$$

$\therefore$ $x_1 = \frac{\pi}{4} - \rho F_x$ 2/

$y_1 = \frac{\pi}{4} - \rho F_y$

$$x_1 = \frac{\pi}{4} - \frac{2}{3}\left(\frac{\pi}{4} - \frac{\sqrt{2}}{2}\right)\left(1 + \frac{\sqrt{2}}{2}\right)$$

$$y_1 = \frac{\pi}{4} - \frac{2}{3}\left(\frac{\pi}{4} - \frac{\sqrt{2}}{2}\right)\left(1 - \frac{\sqrt{2}}{2}\right)$$

même que Newton

Q8.

~~4~~. [20 pts]

$$f(x,y) = \frac{1}{2}(A\vec{x},\vec{x}) \qquad \text{où } \vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}$$

et $A = \begin{pmatrix} 1 & 0 \\ 0 & 10 \end{pmatrix} > 0$

a) [8 pts]

$$x_1 = x_0 + \rho_0 r_x^{(0)} \qquad 2/$$
$$y_1 = y_0 + \rho_0 r_y^{(0)}$$

où $\begin{pmatrix} r_x \\ r_y \end{pmatrix} = b - A\vec{x}_0 = -\begin{pmatrix} 1 & 0 \\ 0 & 10 \end{pmatrix}\begin{pmatrix} 1 \\ 1/10 \end{pmatrix} = -\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\rho_0 = \frac{\|\vec{r}\|^2}{(A\vec{r},\vec{r})} = \frac{2}{\begin{pmatrix} 1 \\ 10 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ 1 \end{pmatrix}} = \frac{2}{11} \qquad 2/$$

$$\Rightarrow \quad x_1 = 1 - \frac{2}{11} = \frac{9}{11}$$

$$2/$$

$$y_1 = \frac{1}{10} - \frac{2}{11} = \frac{-9}{110}$$

b) [6 pts]

On obtient les itérés

$(x_0, y_0) = (1, 1/10)$ — car l'alg. converge en 2 itérations — 2/

$(x_1, y_1) = (9/11, -9/110)$ 2/

$(x_2, y_2) = (0, 0)$ 2/

8

c) [6 pts] ~~facile~~

Posons $\vec{d}_1 = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} - \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} \frac{9}{11} - 1 \\ \frac{-9}{110} - \frac{1}{10} \end{pmatrix}$ 1/

$$\vec{d}_1 = \begin{pmatrix} -2/11 \\ -2/11 \end{pmatrix} \quad 2/$$

et $\vec{d}_2 = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} - \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} \frac{9}{11} \\ \frac{-9}{110} \end{pmatrix}$ 1/

$$\vec{d}_2 = \begin{pmatrix} \frac{-9}{11} \\ \frac{9}{110} \end{pmatrix} \quad 2/$$

On a bien que:

$$(A\vec{d}_1, d_2) \propto \left(A\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ \frac{1}{10} \end{pmatrix}\right)$$

$$= \begin{pmatrix} 1 \\ 10 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ \frac{1}{10} \end{pmatrix} = -1 + 1 = 0$$

Q9.

~~Q5~~ [20 pts]

a) [6 pts]

$$\nabla f(x,y) = \begin{pmatrix} 4x^3 - 4y \\ 4y^3 - 4x \end{pmatrix} = 0 \qquad 1$$

$$\Rightarrow \quad y = x^3 \text{ et } x = y^3$$

$$\Rightarrow \quad x^9 = x \qquad 2$$

$$\text{E.} \quad x(x^8 - 1) = 0$$

$$x(x^2-1)(x^2+1)(x^4+1) = 0$$

$$\Rightarrow x = 0 \text{ ou } x = \pm 1$$

Les pts critiques sont: 3

$(0,0)$, $(1,1)$ et $(-1,-1)$

b) [4 pts]

$$H(x,y) = \begin{pmatrix} 12x^2 & -4 \\ -4 & 12y^2 \end{pmatrix} \qquad 1$$

$$\Rightarrow H(0,0) = \begin{pmatrix} 0 & -4 \\ -4 & 0 \end{pmatrix} \text{ indéfinie} \qquad 1$$

$$H(\pm 1, \pm 1) = \begin{pmatrix} 12 & -4 \\ -4 & 12 \end{pmatrix} > 0 \Rightarrow \text{min. locaux} \qquad 2$$

(c

c) [4 pts]

1)

$$\begin{pmatrix} 12x_k^2 & -4 \\ -4 & 12y_k^2 \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \begin{pmatrix} 4y_k - 4x_k^3 \\ 4x_k - 4y_k^3 \end{pmatrix}$$

1/ 1/ 1/

2)

$$x_{k+1} = x_k + \Delta x$$
$$y_{k+1} = y_k + \Delta y$$

1/

d) [6 pts]

$f(x_0, y_0) = (1/2, 1/2)$

$$\begin{pmatrix} 3 & -4 \\ -4 & 3 \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \begin{pmatrix} 3/2 \\ 3/2 \end{pmatrix}$$ 2/

$\Rightarrow \Delta x = \Delta y = -3/2$ 2/

$x_1 = 1/2 - 3/2 = -1$

$y_1 = \frac{1}{2} - 3/2 = -1$ 1/

$(-1,-1)$ est un min. local 1/

Q10.

~~Q~~4. [20 pts]

a) $f(a,b) = \frac{1}{2}\left[(\overset{0}{\overset{\shortparallel}{1+1-2}})^2 + (e^{a}+e^{-b}-3)^2 + (e^{2a}+e^{-2b}-8)^2 \right]$

$f(a,b) = \frac{1}{2}\left[(e^{a}+e^{-b}-3)^2 + (e^{2a}+e^{-2b}-8)^2 \right]$ 2/

8

$f_a = (e^{a}+e^{-b}-3)e^{a} + 2(e^{2a}+e^{-2b}-8)e^{2a}$ 3/

$= -15e^{2a} + e^{a-b} - 3e^{a} + 2e^{4a} + 2e^{2a-2b}$

$f_b = -(e^{a}+e^{-b}-3)e^{-b} - 2(e^{2a}+e^{-2b}-8)e^{-2b}$ 3/

$= -e^{a-b} - e^{-2b} + 3e^{-b} - 2e^{2a-2b} - 2e^{-4b} + 16e^{-2b}$

b) $F(a,b) = \begin{pmatrix} e^{ax_1} + e^{-bx_1} - y_1 \\ e^{ax_2} + e^{-bx_2} - y_2 \\ \vdots \\ e^{ax_m} + e^{-bx_m} - y_m \end{pmatrix} = \begin{pmatrix} 0 \\ e^{a}+e^{-b} \\ e^{2a}+e^{-2b} \end{pmatrix}$

4

1/ 3/

c)

$a_{k+1} = a_k - \rho f_a(a_k, b_k)$

$b_{k+1} = b_k - \rho f_b(a_k, b_k)$ 2/

4 E1

$a_{k+1} = a_k - \rho\left[(e^{a_k}+e^{-b_k}-3)e^{a_k} + 2(e^{2a_k}+e^{-2b_k}-8)e^{2a_k}\right]$ 2/

$b_{k+1} = b_k - \rho\left[-(e^{a_k}+e^{-b_k}-3)e^{-b_k} - 2(e^{2a_k}+e^{-2b_k}-8)e^{-\ldots}\right]$

/4

d) $\delta = 1/10$

$$a_1 = 1 - \frac{1}{10}\left[(e + e^{-2} - 3)e + 2(e^{2} + e^{-4} - 8)e^{2} \right] 2$$

$$b_1 = 2 - \frac{1}{10}\left[-(e + e^{-2} - 3)e^{-2} - 2(e^{2} + e^{-4} - 8)e^{-4} \right] 2$$

$a_1 =$

$b_1 =$