

# SYSTÈME DE DÉDUCTION NATURELLE

§ 1.4 : Natural deduction [Dirk van Dalen, *Logic and Structure* (3e édition)]

## RÈGLES D'INFÉRENCE

### INTRODUCTION RULES

$$(\wedge I) \quad \frac{\varphi \quad \psi}{\varphi \wedge \psi} \wedge I$$

[ $\varphi$ ]

$$(\rightarrow I) \quad \frac{\begin{array}{c} \vdots \\ \psi \end{array}}{\varphi \rightarrow \psi} \rightarrow I$$

### ELIMINATION RULES

$$(\wedge E) \quad \frac{\varphi \wedge \psi}{\varphi} \wedge E \quad \frac{\varphi \wedge \psi}{\psi} \wedge E$$

$$(\rightarrow E) \quad \frac{\varphi \quad \varphi \rightarrow \psi}{\psi} \rightarrow E$$

We have two rules for  $\perp$ , both of which eliminate  $\perp$ , but introduce a formula.

$$(\perp) \quad \frac{\perp}{\varphi} \perp$$

[ $\neg\varphi$ ]

$$(\text{RAA}) \quad \frac{\begin{array}{c} \vdots \\ \perp \end{array}}{\varphi} \text{RAA}$$

As usual ' $\neg\varphi$ ' is used here as an abbreviation for ' $\varphi \rightarrow \perp$ '.

## EXEMPLES DE DÉRIVATION FORMELLE

$$\text{I} \quad \frac{\frac{\frac{[\varphi \wedge \psi]^1}{\psi} \wedge E \quad \frac{[\varphi \wedge \psi]^1}{\varphi} \wedge E}{\psi \wedge \varphi} \wedge I}{\varphi \wedge \psi \rightarrow \psi \wedge \varphi} \rightarrow I_1$$



$$\text{II} \quad \frac{\frac{\frac{[\varphi]^2 \quad [\varphi \rightarrow \perp]^1}{\perp} \rightarrow E}{(\varphi \rightarrow \perp) \rightarrow \perp} \rightarrow I_1}{\varphi \rightarrow ((\varphi \rightarrow \perp) \rightarrow \perp)} \rightarrow I_2$$

$$\text{II}' \quad \frac{\frac{\frac{[\varphi]^2 \quad [\neg\varphi]^1}{\perp} \rightarrow E}{\neg\neg\varphi} \rightarrow I_1}{\varphi \rightarrow \neg\neg\varphi} \rightarrow I_2$$



$$\text{III} \quad \frac{\frac{\frac{\frac{[\varphi \wedge \psi]^1}{\psi} \wedge E \quad \frac{[\varphi \wedge \psi]^1}{\varphi} \wedge E}{\psi \rightarrow \sigma} \rightarrow E}{\sigma} \rightarrow I_1}{\varphi \wedge \psi \rightarrow \sigma} \rightarrow I_2}{\varphi \rightarrow (\psi \rightarrow \sigma)} \rightarrow I_2$$

